

Exam 1
Chapter 1 and 2.1

Name: _____

Do not write your name on any other page. Answer the following questions. *Answers without proper evidence of knowledge will not be given credit.* Make sure to make reasonable simplifications. Do not approximate answers. Give exact answers. **Only scientific calculators are allowed on this exam.**

Show your work!

1. (5 points) Verify that $y = x \cos x$ is a solution to the differential equation

$$y' + y \tan x = \cos x.$$

2. (5 points) A diesel car gradually speeds up so that for the first 10 s its acceleration is given by

$$\frac{dv}{dt} = (0.12)t^2 + (0.6)t \quad (\text{ft/s}^2).$$

If the car starts from rest ($x_0 = 0$, $v_0 = 0$), find the distance it has traveled at the end of the first 10 seconds and its velocity at that time.

3. (10 points) Find the general solution to the differential equation

$$2\sqrt{x}\frac{dy}{dx} = \cos^2 y.$$

4. A tank initially contains 60 gallons of pure water. Brine containing 1 lb of salt per gallon enters the tank at a rate of 2 gal/min, and the (perfectly mixed) solution leaves the tank at 3 gal/min.

- (a) (2 points) When is the tank empty?
- (b) (6 points) Find the amount of salt in the tank after t minutes.
- (c) (2 points) What is the maximum amount of salt ever in the tank?

5. (10 points) Find the general solution to the differential equation

$$(1 + ye^{xy})dx + (2y + xe^{xy})dy = 0.$$

6. (10 points) Consider a rabbit population satisfying the logistic equation

$$\frac{dP}{dt} = 2P - (0.005)P^2.$$

If the initial population is 120 rabbits, how many months does it take for $P(t)$ to reach 95% of its limiting population M ?